Answers to Exam for Dynamical Systems and Chaos, 5 April 2017.

I.

1. The fixed point is $(x^*, y^*) = (0,0)$.
2. $$J = \left\{ \begin{array}{cc} -1+y & 1+x \\ 1-2x & -1-3y^2 \end{array} \right\} \quad (1)$$
3. For $(x^*, y^*) = (0,0), \lambda = -2, 0,\ v_1 = (1,1),\ v_2 = (1,-1)$.
4. Stable in one direction, but marginal in the other.
5. $V(x,y) = ax^2 + 2y^2 \Rightarrow \dot{V}(x,y) = 2ax\dot{x} + 4y\dot{y} = 2ax(-x+y+yx) + 4y(x-y-x^2-y^3) = -2ax^2 + 2axy + 2ax^2y + 4xy - 4y^2 - 4yx^2 - 4y^4 =\ (a=2) 8xy - 4y^2 - 4x^2 - 4y^4 = -4(x-y)^2 - 4y^4 < 0$ if $(x^*, y^*) \neq (0,0)$. Therefore $(x^*, y^*) = (0,0)$ is stable.
6. From above limit cycle is not possible.

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7. Null clines: $\dot{x} = 0$ for $x = 0, y = x(1-x)$ and $\dot{y} = 0$ for $y = 0, x = a$
8. $(x^*, y^*) = (0,0), (1,0), (a, a-a^2)$
9. For the fixed point $(x^*, y^*) = (1,0)$ the predator is extinct while prey exists.
10. $$J = \left\{ \begin{array}{cc} 2x - 3x^2 - y & -x \\ y & x-a \end{array} \right\} \quad (2)$$

    $(x^*, y^*) = (0,0), \lambda = -a, 0$, one direction stable, one marginal.

    $(x^*, y^*) = (1,0), \lambda = -1, 1-a$, one direction stable, one unstable, saddle point.
11. $(x^*, y^*) = (a, a-a^2) \Rightarrow \tau = -2a^2 + a, \Delta = -a^3 + a^2$

    The Hopf occurs when $\tau = a - 2a^2 = 0 \Rightarrow a_x = \frac{1}{2}$ while $\Delta > 0$, Change from complex stable to complex unstable.
12. For $0 < a < \frac{1}{2} \Rightarrow \tau > 0$ so unstable.

III.

13. $f'(x_n) = (1 - rx_n)exp(r(1-x_n))\ x_m = \frac{1}{r}$, minimum for $r < 0$, maximum for $r > 0$
14. $x^* = 0, 1$
15. $f'(0) = exp(r) \Rightarrow$ unstable for $\mathrm{r} > 0$, stable for $\mathrm{r} < 0$

    $f'(1) = 1 - r \Rightarrow$ unstable for $\mathrm{r} < 0$ and $\mathrm{r} > 2$
16. $f'(1) = 1 - r = -1 \Rightarrow r_p = 2$
17. $x_m = \frac{1}{r} \rightarrow \frac{1}{r}exp(r-1) \rightarrow \frac{1}{r}exp(r-1)exp(r - exp(r-1)) = \frac{1}{r} \Rightarrow xp(r-1)exp(r - exp(r-1)) = 1 \Rightarrow 2r - 1 - exp(r-1) = 0$
18. The value $r = r_s = 1$ is a solution, the superstable fixed point is also a two-cycle.