Spørgsmål 1-14 indgår med lige vægt i bedømmelsen.
Eksamensresultat og løsninger opslås på hjemmesiden.

I.

Consider the system of differential equations

$$\begin{aligned} \dot{u} &= au - bvu \\ \dot{v} &= -v + du^2 \end{aligned} \tag{1}$$

where a, b, d are constants.

1. Show that there exists a re-scaling transformation $u = Ax$ and $v = By$ so (1) can be brought on the form

$$\begin{aligned} \dot{x} &= ax - xy \\ \dot{y} &= -y + x^2 \end{aligned} \tag{2}$$

   and derive $A(b,d)$ and $B(b,d)$.

2. Find the fixed points of (2).
3. Determine the stability of the fixed points.
4. What kind of bifurcation takes place as a is varied ? Sketch the bifurcation diagram.
5. What happens for $a > 1/8$?

II.

Consider the system of differential equations

$$\begin{aligned} \dot{x} &= y + axy^2 \\ \dot{y} &= -x + yx^2 \end{aligned} \tag{3}$$

6. We define a symmetry transformation by R by $R(x,y) = (y,x)$. Show that the system is reversible (i.e. invariant under $(x,y) \to R(x,y)$, $t \to -t$) for one value of a and determine this value.
7. For this value of a, show that there are invariant curves around $(0,0)$.
8. The system (3) has lines of fixed points for one value of a. Determine this value of a and the line of fixed points.
9. For the value of a determined in 8, draw characteristic phase portraits in the (x,y)-plane, including orbits through (1,0) and (2,0). Find, for the case $x(0) = 2$, $y(0) = 0$ the limit $x^* = \lim_{t\to\infty} x(t)$.

III.

Consider the discrete mapping

$$x_{n+1} = f(x_n) = \begin{cases} 3x_n & \text{if } 0 \le x_n \le 1/3 \\ \frac{3}{2}(1 - x_n) & \text{if } 1/3 \le x_n \le 1 \end{cases} \tag{4}$$

10. Determine the fixed points of (4) and their stability.
11. Find the orbit points of the two-cycle (where $x_n^* = f^2(x_n^*)$). Determine the stability of the two-cycle.

    A cycle-point x_n^* can be associated with the letter L if $x_n^* \in [0;1/3]$ and can be associated with the letter R if $x_n^* \in [1/3;1]$. In terms of these symbols, the two-cycle can under the mapping (4) be described as $LRLRLR$.....

12. The mapping (4) has two three-cycles, where $x_n^* = f^3(x_n^*)$ (and different from the fixed point). Write, in terms of the symbols L and R, the two possible sequences for the three-cycles.
13. Determine the orbit points of one of the three-cycles.
14. Describe the stability of all periodic cycles of the mapping (4).