Dynamical Systems and Chaos 2015 Spring

Homework Solutions, Session 13

March 11, 2015

8 Bifurcations Revisited

8.2 Hopf Bifurcations

8.2.3

Subcritical Hopf bifurcation

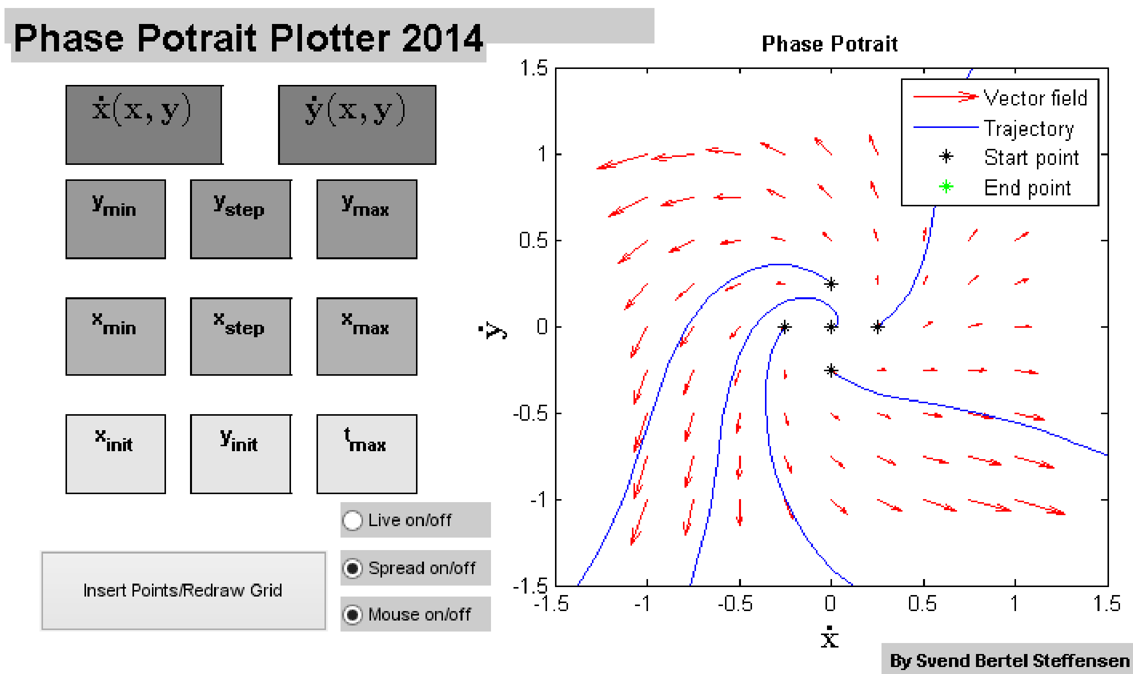

Figure 1: 8.2.3. $\mu = 1$

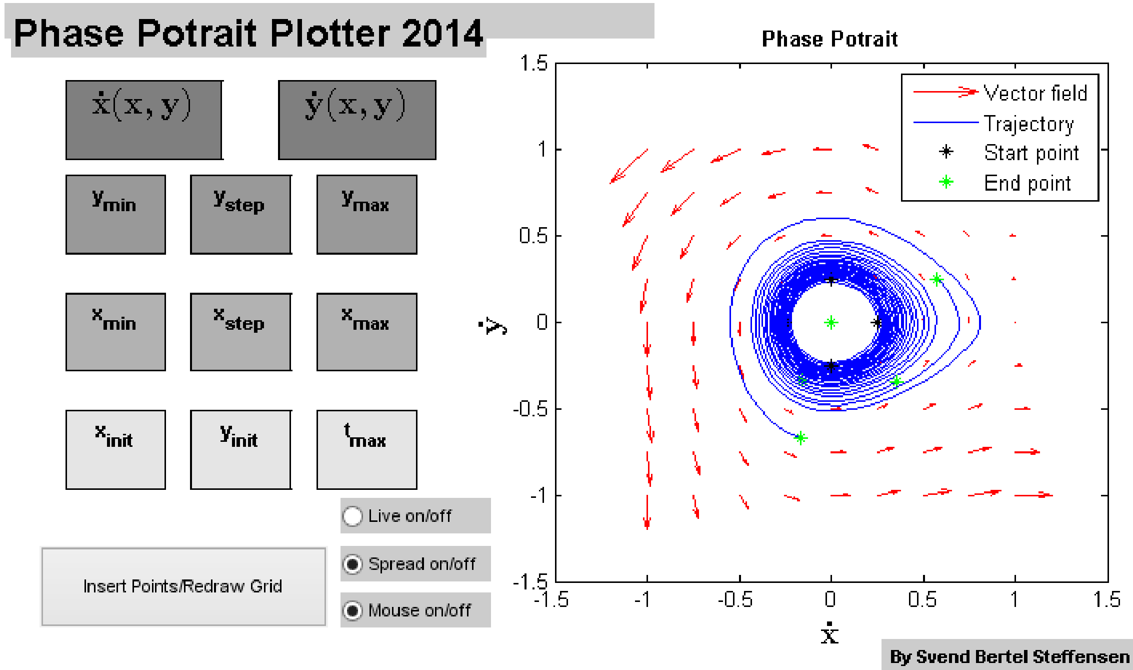

Figure 2: 8.2.3. $\mu = 0$

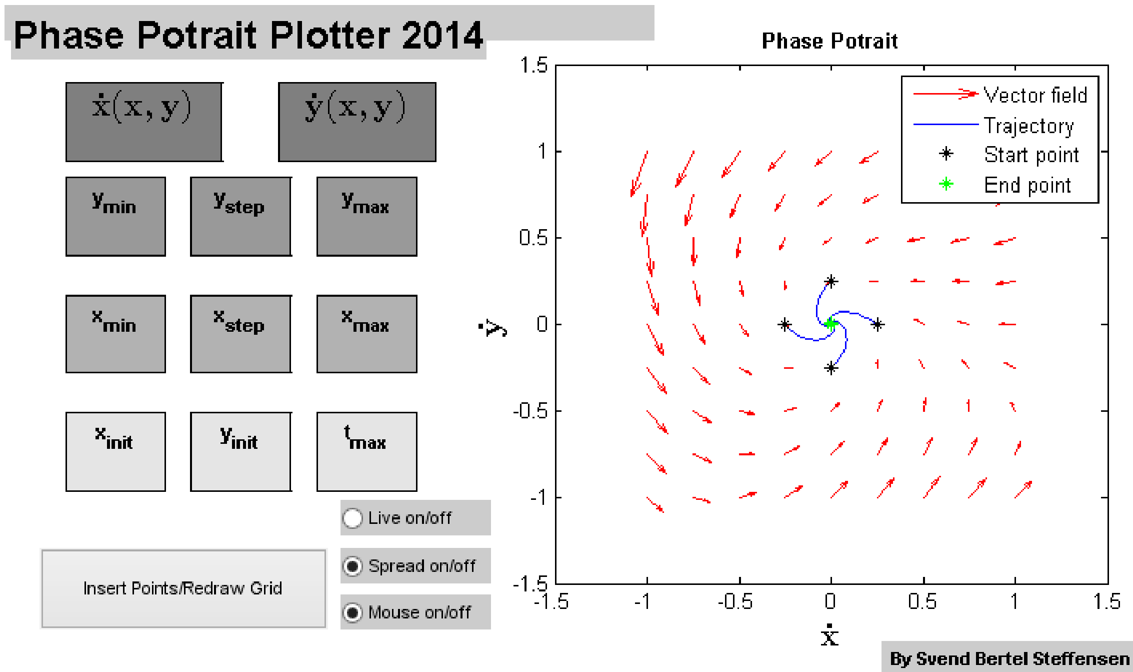

Figure 3: 8.2.3. $\mu = -1$

11 Fractals

11.3 Dimension of Self-Similar Fractals

11.3.8

(b) $r = 3$ and $m = 8$. Therefore the dimension is $d = \ln 8/\ln 3$.

(c) The available fraction of area after n removals is $A_n = (8/9)^n$, so the removed area at $(n+1)$-th removal is $r_{n+1} = (1/9)A_n = (1/9)(8/9)^n$. Therefore, the total removed area is

$$R = \sum_{n=1}^{\infty} r_n = \frac{1}{9}\sum_{n=1}^{\infty}\left(\frac{8}{9}\right)^{n-1} = \frac{1}{9}9 = 1$$

Therefore, the Sierpinski carpet has zero area.

11.4 Box Dimension

11.4.2

During each partition, the length of the box is reduced by 3-fold and each original box creates 8 new boxes of smaller sizes. Therefore, $\epsilon_n = 1/3^n$ and $N_n = 8^n$, and the dimension is $d = \lim_{n\to\infty} \ln(N_n)/\ln(1/\epsilon_n) = \ln(8)/\ln(3)$.