Dynamical Systems and Chaos

Mathias Heltberg, Chengzhe Tian

Exercise Session #01

2015.02.04

Contents

Administrative Issues

- Danish Tutorial Session (Hold 1)
  - Mathias Heltberg
    - PhD student, Biocomplexity, NBI
    - Email: mathiasheltberg@hotmail.com
    - Office: 07-1-kb4
  - Time
    - Monday 14:15-16:00, A105 HCO
    - Wednesday 13:15-15:00, 1-0-10 DIKU
- English Tutorial Session (Hold 2)
  - Chengzhe Tian
    - PhD student, Biocomplexity, NBI
    - Email: chengzhe@nbi.dk
    - Office: 07-1-kb3
  - Time
    - Monday 14:15-16:00, 1-0-04 DIKU
    - Wednesday 13:15-15:00, S15 HCO

Plan

- Every Wednesday, a short recap of course materials (10 min)
- A list of mathematical tricks is provided.
- The solution of exercises will be uploaded to the course website after each exercise session.
- Danish Tutorial (From the second week)
  - Solve exercises before class
  - Present solutions by students in class
- English Tutorial (From the second week)
  - Solve and present around 1/3 exercises in class
  - Refer to online solutions for the remaining exercises

Reminder

Please frequently check the course website
dynamical-systems.nbi.dk

Contents

Dynamical System

Definition

A dynamical system is defined by the (semi-)flow

$$\frac{\mathrm{d}x_1}{\mathrm{d}t} = f_1(x_1, x_2, \ldots, x_n)$$

$$\vdots$$

$$\frac{\mathrm{d}x_n}{\mathrm{d}t} = f_n(x_1, x_2, \ldots, x_n)$$

where $x_1, \ldots, x_n \in \mathbb{F}$, $f_1, \ldots, f_n \in \mathbb{C}_{\mathbb{F}}$ and $t \in \mathbb{R}_{\geq 0}$.
$x_1, \ldots, x_n$: states
n: dimension of the system

Representing Higher Order

For a system

$$B_n x^{(n)} + B_{n-1} x^{(n-1)} + \cdots + B_1 \dot{x} + B_0 x + C = 0$$

where $x^{(n)} = \mathrm{d}^n x / \mathrm{d}t^n$.

Representing Higher Order

For a system

$$B_n x^{(n)} + B_{n-1} x^{(n-1)} + \cdots + B_1 \dot{x} + B_0 x + C = 0$$

where $x^{(n)} = \mathrm{d}^n x / \mathrm{d}t^n$.
Define

$$x_1 = x; x_2 = \dot{x} = \dot{x}_1; \ldots; x_n = x^{(n-1)} = \dot{x}_{n-1}$$

and we have

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ &\vdots \\ \dot{x}_{n-1} &= x_n \\ \dot{x}_n &= -\frac{B_{n-1}x_n + \cdots + B_1 x_2 + B_0 x_1 + C}{B_n} \end{aligned}$$

Classification of Dynamical System

Classification

A dynamical system is *linear* if all the terms are linear with respect to the state variables x_i, $i = 1, 2, \ldots, n$. Otherwise it's *nonlinear*.

Classification of Dynamical System

Classification

A dynamical system is *linear* if all the terms are linear with respect to the state variables x_i, $i = 1, 2, \ldots, n$. Otherwise it's *nonlinear*.

Classify the following ODE: Linear or Nonlinear?

- $\dot{x}_1 = x_2, \dot{x}_2 = -x_1$
- $\dot{x}_1 = \alpha - \beta x_1 x_2, \dot{x}_2 = \beta x_1 x_2 - \gamma x_2$
- $\dot{x}_1 = \sin^3 t$
- $\dot{x}_1 = x_2 \sin t; \dot{x}_2 = x_1 \cos t$
- $\dot{x}_1 = x_1(\sin t + \cos^2 t)$

Classification of Dynamical System

Classification

A dynamical system is *linear* if all the terms are linear with respect to the state variables x_i, $i = 1, 2, \ldots, n$. Otherwise it's *nonlinear*.

Classify the following ODE: Linear or Nonlinear?

- $\dot{x}_1 = x_2, \dot{x}_2 = -x_1$
- $\dot{x}_1 = \alpha - \beta x_1 x_2, \dot{x}_2 = \beta x_1 x_2 - \gamma x_2$
- $\dot{x}_1 = \sin^3 t$
- $\dot{x}_1 = x_2 \sin t; \dot{x}_2 = x_1 \cos t$
- $\dot{x}_1 = x_1(\sin t + \cos^2 t)$

Solution: Linear, Nonlinear, Linear, Linear, Linear

Flow on 1D

1D Flow

$$\dot{x} = f(x)$$

where $x \in \mathbb{F}, f \in \mathbb{C}_{\mathbb{F}}$ and $t \in \mathbb{R}_{\geq 0}$

Note: we ignore the case where $f = f(x, t)$, though it is a 1D flow.

Fix Points (Intuitively)

Fix points: x doesn't change its position at this point.
$\dot{x} = f(x) = 0$.
Stable fix points: when x is a little bit away from the fix point, it will go back.
Unstable fix points: when x is a little bit away from the fix point, it will go away.

Concrete definition in Chapter 2.4 (Linear Stability Analysis).

Contents

Exercises

Please solve exercise 2.1.1-2.1.4, 2.2.1, 2.2.5, 2.2.11, 2.3.2 first.