Dynamical Systems and Chaos

Excersize class

11/2-2015

Hints for the excersizes

1) 4.3.1
We want to find a solution to the integral

$$\int \frac{1}{r + x^2} \tag{1}$$

For this type it is always smart to use the substitution $x = \sqrt{r}tan(\theta)$
Now remember that

$$\frac{dx}{d\theta} = \sqrt{r}(1 + tan(\theta)^2) \tag{2}$$

With this at hand we can solve the integral by the substitution. In the end we need to change the limits. Remember that $tan(-\pi/2) = -\infty$ and $tan(\theta/2) = \infty$.
2) 5.1.1
Following the hint of the book we can divide the two to get:

$$\frac{dx}{dv} = \frac{v}{-\omega^2 x} \tag{3}$$

Now putting all v's on one side, and all x's on the other we can solve get the expression. Then remember the terms for kinetic and potential energy for a harmonic oscillator.
3) 5.1.9
Start by skething the vector field as described in example 5.1.1 and follow the technique used in the excersice 5.1.1. The stable manifold is now the line on which the flow goes towards the fixpoint
4) 5.2.4
Write up the two equations and the matrix A. Now find the eigenvalues where we can use:

$$\lambda_{1,2} = \frac{\tau \pm \sqrt{\tau^2 - 4\Delta}}{2} \quad \text{See 5.2(4-5) in the book} \tag{4}$$

Then find the eigenvectors from:

$$\begin{pmatrix} a-\lambda & b \\ c & d-\lambda \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad \text{See example 5.2.1!} \tag{5}$$

Then remember we can classify the fixpoints from figure 5.2.8 (which from now on should be our favourite figure in the course).
5) 5.2.2
Use the technique from excersize 5.2.4 and find the eigenvalues and eigenvectors. Remember that the general solution of $\bar{x}$ is:

$$\mathbf{x}(t) = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = c_1 e^{\lambda_1 t} \begin{pmatrix} v_{1,1} \\ y_{1,2} \end{pmatrix} + c_2 e^{\lambda_2 t} \begin{pmatrix} v_{2,1} \\ y_{2,2} \end{pmatrix} \tag{6}$$

6) 5.2.11

Remember we can write the characteristic equation as:

$$(a-\lambda)(d-\lambda)-bc=0 \tag{7}$$

So if a=d and bc=0 you can show that there is only one eigenvalue. Next we need to solve the system which can be written as:

$$\begin{aligned}\dot{x} &= \lambda x - by \\ \dot{y} &= \lambda y\end{aligned} \tag{8}$$

The equation for $\dot{y}$ is a simple differential equation that is easily solved. This could then be inserted the equation for $\dot{x}$. This equation can now take the form:

$$\dot{x}-\lambda x = y(t). \tag{9}$$

The strategy is now to find a function $\mu(t)$ (as usual it might be clever to start by considering the exponential function) we can multiply on both sides so we can rewrite the equation as:

$$\frac{d}{dt}(\mu(t)x(t)) = \mu(t)y(t) \quad \text{See Mathematical Methods... section 14.2.4} \tag{10}$$

Then we can integrate this to find x(t) and draw the phase plot.