Spørgsmål 1-15 indgår med lige vægt i bedømmelsen.
Svarene kan skrives på dansk og/eller engelsk. Blyant er tilladt.
Eksamensresultat og løsninger opslås på hjemmesiden.

I.

Consider the system of differential equations

$$\begin{aligned} \dot{x} &= x^2 - y - 1 \\ \dot{y} &= (x-2)y \end{aligned} \tag{1}$$

1. Find the fixed point(s) for the system (1).
2. Derive the Jacobian matrix for the system (1).
3. Determine the nature and the stability of all the fixed points.
4. Draw the nullclines of the system (1).
5. Sketch the phase portrait and the flow for the system (1).

II.

Consider the differential equation

$$\dot{x} = f(x,r) = r\ x\ -\ 2x^2\ +\ x^3\ \ ,\qquad r \in]-\infty;\infty[\tag{2}$$

6. Find the fixed points for (2) (some of the fixed points as a function of r).
7. Identify a bifurcation point and find the critical value of the parameter, r_c.
8. What type of bifurcation takes place at r_c.
9. Determine the stability of the fixed points for (2).
10. Sketch the bifurcation diagram, i.e. fixed points x^* as a function of r (full line: stable fixed points, dashed line: unstable fixed points).

III.

Consider the discrete mapping

$$x_{n+1} = f(x_n) = \begin{cases} 3x_n & \text{if} \quad x_n \leq 1/2 \\ 3 - 3x_n & \text{if} \quad x_n \geq 1/2 \end{cases} \tag{3}$$

11. Determine the fixed points for (3) and their stability (hint: draw $f(x_n)$ to get intuition).
12. Find the orbit points of the 2-cycle (where $x^* = f(f(x^*))$). Note that one point of the 2-cycle is less than 1/2 and the other point is greater than 1/2 (hint: draw the cobweb for a 2-cycle of $f(x_n)$ to get intuition).
13. Determine the stability of the two-cycle.
14. Show that the point x_n^* =3/28 is an orbit point of a 3-cycle for $f(x_n)$ (a 3-cycle fulfills $x^* = f(f(f(x^*)))$). Determine the other two orbit points on this 3-cycle.
15. What do you think will happen to the iteration sequence x_n (as $n \to \infty$) if the initial point x_0 lie in the interval $x_0 \in]1/3; 2/3[$.