I.

1. $(x^*, y^*) = (0,0), (\frac{3}{2}, 0), (0, \frac{4}{3}), (1,1)$.

2. $$J = \left\{ \begin{array}{cc} 3-4x-y & -x \\ -y & 4-x-6y \end{array} \right\} \tag{1}$$

3. $(0,0), \lambda = 3, 4, \tau = 7, \Delta = 12$: unstable node
   $(0, \frac{4}{3}), \lambda = \frac{5}{3}, -4, \tau = -\frac{7}{3}, \Delta = -\frac{20}{3}$: saddle
   $(\frac{3}{2}, 0), \lambda = \frac{5}{2}, -3, \tau = -\frac{1}{2}, \Delta = -\frac{15}{2}$: saddle
   $(1,1), \lambda = \frac{-5 \pm \sqrt{5}}{2}, \tau = -5, \Delta = 5$: stable node

4. Eigendirections for $(1,1) : (x,y) = (1, \frac{1-\sqrt{5}}{2}), (x,y) = (1, \frac{1+\sqrt{5}}{2})$

5. Null-clines: $x = 0, y = 0, y = 3 - 2x, y = \frac{4}{3} - \frac{1}{3}x$

6. $(1,1)$ stable, arrows point to it; $(0,3)$ saddle: stable and unstable manifold; $(4,0)$ saddle: stable and unstable manifold; $(0,0)$: ustable node, arrows point away from it. The fixed point $(1,1)$ is stable so species x and y are able to live together both with a population of the same size.

II.

7. $x^* = rx^* + \frac{1}{x^*} \Rightarrow x^* = \pm\frac{1}{\sqrt{1-r}}$

8. $f'(x) = r - \frac{1}{x^{*2}} \Rightarrow f'(x^*) = 2r - 1$ $f'(x^*) = 1 \Rightarrow r_{c1} = 1; f'(x^*) = -1 \Rightarrow r_{c2} = 0$ So fixed points stable when $0 < r < 1$.

9. $f'(x^*) = 0 \Rightarrow r_s = \frac{1}{2}$. The super stable fixed point $x_s = \sqrt{2}$.

10. Two cycle: $x \to rx + \frac{1}{x} \to r(rx + \frac{1}{x}) + \frac{1}{rx + \frac{1}{x}} = x \Rightarrow (r^3 - r)x^4 + 2r^2x^2 + r = 0 \Rightarrow x^2 (= y) = \frac{-r \pm 1}{r^2 - 1}$

11. For $r = -2$ $x = \pm 1$ is a solution to the two-cycle: For $r = -2$ we see: $1 \to -1 \to 1....$ Stability: $f'(x) = r - \frac{1}{x^{*2}} \Rightarrow f'(1) \cdot f'(-1) = (-3) \cdot (-3) = 9 > 1$: unstable.

III.

12. $x^* = 1, x^* = \frac{-1 \pm \sqrt{1-4r}}{2}$.

13. A bifurcations takes place at $r_c = \frac{1}{4}$

14. A saddle-node bifurcations takes place: The hyperbola 'goes through' the parabola. Just after the bifurcation $(0 < r < \frac{1}{4})$ the hyperbola is 'above' the parabola. Therefore the '-' branch is stable whereas the '+' branch is unstable.

15. Saddle-node bifurcation (out of the blue sky).

16. Saddle-node bifurcation takes place at $r_c = \frac{1}{4}$, upper branch '+' unstable, lower branch '-' stable. However: the bifurcated fixed points change stability at $r = 0$. For $r < 0$: upper branch '+' stable, lower branch '-' unstable. $x^* = 1$ is stable below $r = -2$ and unstable above. The '+' branch meet $x^* = 1$ for $r = -2$ and change stability. Therefore a transcritical bifurcations takes place here.