Spørgsmål 1-16 indgår med lige vægt i bedømmelsen.
Svarene kan skrives på dansk og/eller engelsk. Blyant er tilladt.
Eksamensresultat og løsninger opslås på hjemmesiden.

I.

Consider the system of differential equations

$$\begin{aligned} \dot{x} &= x(3-2x-y) \\ \dot{y} &= y(4-x-3y) \end{aligned} \tag{1}$$

1. Find the fixed point(s) for the system (1).
2. Derive the Jacobian matrix for the system (1).
3. Determine the nature and the stability of all the fixed points.
4. Determine all the eigendirections.
5. Draw the nullclines of the system (1). Sketch the phase portrait and the flow for the system (1).
6. Let x represents the number of a given biological species and y represents the number of another biological species, and assume they live on a selected, limited area. What conclusions can you draw from the results of the phase plane dynamics.

II.

Consider the differential equation

$$\dot{x} = f(x,r) = x^2 + r - 1 - \frac{r}{x}, \; x \neq 0 \tag{2}$$

7. Find the fixed points for (2) as a function of r. Hint: Note that $x^* = 1$ is always a fixed point.
8. Determine the stability of the fixed points for (2).
9. Let $g(x) = x^2 + r - 1$ and $h(x) = \frac{r}{x}$. By drawing the two functions $g(x), h(x)$ for r-values around r_c, sketch how the bifurcation takes place in the plane. Indicate on the sketch the stability of the fixed points.
10. Identify a bifurcation point and find the critical value of the parameter, r_c.
11. What type of bifurcation takes place at r_c.
12. Sketch the bifurcation diagram, i.e. fixed points x^* as a function of r (full line: stable fixed points, dashed line: unstable fixed points).

III.

Consider the discrete mapping

$$x_{n+1} = f(x_n) = rx_n + \frac{1}{x_n}, \quad x_n \neq 0 \tag{3}$$

13. Determine the fixed points for (3) and their stability.
14. Determine the value of the parameter r_s where the fixed point is super-stable. Determine the super stable fixed point x_s.
15. Determine the values of the parameter $r_{c1} > r_{c2}$ where the fixed points become unstable.
16. Write the equation in x and r for the two cycle (Hint: Set $y = x^2$). and solve the equation for x to find the cycle points.
17. For $r = -2$, find (or guess!) the cycle points. What is the stability of the two-cycle at this point.