I.

1. $(x^*, y^*) = (2,3), (1,0), (-1,0)$.

2. $$J = \left\{ \begin{array}{cc} 2x & -1 \\ y & x-2 \end{array} \right\} \tag{1}$$

3. $(2,3), \tau = 4, \Delta = 3, \lambda = 1, 3$: unstable node

   $(1,0), \tau = 1, \Delta = -2, \lambda = 2, -1$: saddle

   $(-1,0), \tau = -5, \Delta = 6, \lambda = -2, -3$: stable node

4. Null-clines: $y = 0, x = 2, y = x^2 - 1$

5. $(-1,0)$ stable, arrows point to it; $(1,0)$ saddle: stable and unstable manifold; $(2,3)$: unstable, arrows point away.

II.

6. $x^* = 1 \pm \sqrt{1-r^2}$. They exist only in the interval $-1 \leq r \leq 1$.

7. Bifurcations take place at $r_{c1} = 1$ and $r_{c2} = -1$.

8. Both are saddle-node bifurcation (out of the blue sky).

9. $f'(x) = -2 + 2x$

   $f'(1 \pm \sqrt{1-r^2}) = \pm 2\sqrt{1-r^2}$

   The '+' branch is unstable.

   The '-' branch is stable.

10. Saddle-node bifurcation takes place at $r_{c1} = 1$ (where $x = 1$) and at $r_{c2} = -1$ (where also $x = 1$). The 'upper' branch is unstable (dashed line) (it goes through the point (0,2). The 'lower' branch is stable (full line) (it goes through the point (0,0)). The upper and lower branches close at each other (forming a 'ring').

III.

11. $x^* = 0, \frac{3}{4}$. The slope in any point is $f'(x) = |3|$ and therefore they are always unstable.

12. The 2-cycle fulfills: $x \to 3x \to 3 - 3(3x) \;=\; x \;\;\Rightarrow\;\; x = \frac{3}{10}$. The other orbit point is therefore $\frac{9}{10}$.

13. The slope in any point is $f'(x) = |3|$ and therefore the 2-cycle is unstable.

14. Starting out in $\frac{3}{28}$ this 3-cycle must fulfill: $\frac{3}{28} \to \frac{9}{28} \to \frac{27}{28} \to 3 - 3\frac{27}{28} = \frac{3}{28}$. We see it comes back to $\frac{3}{28}$ after three iterations.

15. The first iterate of a point $x_0 \in]1/3; 2/3[$ is larger than 1, and it will therefore diverge versus $\pm\infty$.