Spørgsmål 1-14 indgår med lige vægt i bedømmelsen.
Eksamensresultat og løsninger opslås på hjemmesiden.

I.

Consider the system of differential equations

$$\begin{aligned} \dot{x} &= y(x+1) \\ \dot{y} &= x(1+y^3) \end{aligned} \tag{1}$$

1. Find the fixed points of the system.
2. Determine the type and stability of the fixed points.
3. Sketch the phase plane of the system (e.g. nullclines, eigendirections, approximate flow lines).

II.

Consider the system of differential equations

$$\begin{aligned} \dot{x} &= -x - 2y^2 \\ \dot{y} &= xy - x^2 y \end{aligned} \tag{2}$$

4. Show that the system has only one fixed point.
5. Construct a Liapunov function $V(x,y)$ to show that the fixed point is stable (Hint: Use e.g. $V(x,y) = x^2 + ay^2$, where a is a parameter).

III.

Consider Rayleigh's differential equation

$$\ddot{x} + \epsilon(\frac{1}{3}\dot{x}^3 - \dot{x}) + x = 0 \tag{3}$$

where ϵ is small and positive.

6. Use averaging theory to derive an equation for r'.
7. Use averaging theory to derive an equation for ϕ'.
8. Solve the equation for r' with the boundary conditions $x(0) = \sqrt{2}$, $\dot{x}(0) = 0$ (Hint: see Strogatz p.225).

IV.

The Hamiltonian for the positions ϕ_n of particles connected with springs and each subjected to a ϕ^4 potential is given by $(a > 0)$

$$H = \sum_n \frac{1}{2}(\phi_{n+1} - \phi_n)^2 + \frac{1}{4}a(\phi_n^2 - 1)^2 \tag{4}$$

9. Show that the equilibrium positions determined by $\frac{\partial H}{\partial \phi_n} = 0$ will satisfy the relation $\phi_{n+1} + \phi_{n-1} - 2\phi_n = a\phi_n(\phi_n^2 - 1)$.
10. Show that by the substitution $x_n = \phi_n$, $y_n = \phi_{n-1}$, this relation can be transformed into a two-dimensional mapping $(x_{n+1}, y_{n+1}) = T(x_n, y_n)$ on the form

$$T(x_n, y_n) = (2x_n + ax_n(x_n^2 - 1) - y_n, x_n) \tag{5}$$

11. Show that T is area-preserving.
12. Find the fixed points for T.
13. Determine the eigenvalues and the stability of the fixed points.
14. Show that the fixed point $(0,0)$ bifurcates at $a = 4$. What type of bifurcation is that ?