**Løsninger til Dynamiske Systemer og Kaos, 9. april 2008.**

## I.

1. $x^* = \frac{r \pm \sqrt{r^2+4r}}{2}$.
2. $r_{c1} = -4, r_{c2} = 0$
3. $f'(x,r) = 2x - r = \pm\sqrt{r^2+4r}$ The '+' solution is unstable, the '-' solution is stable
4. Saddle-node bifurcations both.
5. $\sqrt{r^2+4r} = r\sqrt{1+4/r} \simeq r(1+2/r) = r+2$ The asymptotic behavior for $r \to \infty$: $x^* = \frac{r \pm (r+2)}{2} = r$ or $-1$
6. Bifurcations happen at $r_{c1} = -4$, $x^* = -2$ and $r_{c2} = 0$, $x^* = 0$. Upper branches unstable. Lower branches stable.

   For large, positive $r$: Upper branch approaches the straight line $x^* = r$. Lower branch approaches the line $x^* = -1$.

## II.

7. $x_n^* = 0, \frac{2}{5}, \frac{4}{3}$. All derivatives are $\pm 4$ so they are all unstable.
8. Under the mapping: $x \to 4x \to 2 - 16x = x \Rightarrow \frac{2}{17} \to \frac{8}{17} \to \frac{2}{17}$
9. Stability: $f'(\frac{2}{17}) \cdot f'(\frac{8}{17}) = -16$, unstable
10. Under the mapping: $x \to 4x \to 16x \to 2 - 64x = x \Rightarrow \frac{2}{65} \to \frac{8}{65} \to \frac{32}{65} \to \frac{2}{65}$.
11. $x_0 > 4/3$.

## III.

12. $r'(T) =< -r^5 cos^4\theta sin^2\theta + arsin^2\theta >= -\frac{1}{16}r(r^4 - 8a)$

    $\phi'(T) =< -r^5 cos^5\theta sin\theta + arsin\theta cos\theta >= 0$
13. $r'(T) = 0 \Rightarrow r^* = 0$ or $r^* = (8a)^{1/4}$
14. $\frac{dr'(T)}{dr} = -\frac{5}{16}r^4 + \frac{1}{2}a$

    Inserting $r^* = (8a)^{1/4}$ : $\frac{dr'(T)}{dr} = -2a$ and $r^* = (8a)^{1/4}$ is therefore stable.

    Inserting $r^* = 0$ : $\frac{dr'(T)}{dr} = \frac{1}{2}a$ and $r^* = 0$ is therefore unstable.
15. $a = 2 \Rightarrow \frac{dr}{dT} = \frac{1}{16}r(16 - r^4) \Rightarrow \frac{16dr}{r(16-r^4)} = dT \Rightarrow dr(\frac{1}{r} + \frac{r^3}{16-r^4}) = dT$.

    Integration gives: $lnr \; - \; \frac{1}{4}ln(16 - r^4) \; = \; T + C$

    From initial condition: $r(0) = 1 \Rightarrow C = -\frac{1}{4}ln(15)$

    Inserting and after some algrbra one finds:

    $r(T) = \frac{2}{(1+15e^{-4T})^{1/4}}$

    Initial point and asymptotes: $r(0) = 1, r(T) \to 2$ for T $\to \infty$